

FIITJEE Solutions to IIT-JEE-2010

PAPER 2

CODE

0

Time: 3 Hours

Please read the instructions carefully. You are allotted 5 minutes specifically for this purpose.

INSTRUCTIONS

A. General:

1. This Question Paper contains 32 pages having 57 questions.
2. The **question paper CODE** is printed on the right hand top corner of this sheet and also on the back page (page no. 32) of this booklet.
3. No additional sheets will be provided for rough work.
4. Blank papers, clipboards, log tables, slide rules, calculators, cellular phones, pagers and electronic gadgets in any form are not allowed.
5. Log and Antilog tables are given in page numbers 30 and 31 respectively.
6. The answer sheet, a machine-gradable Objective Response Sheet (**ORS**), is provided separately.
7. Do not Tamper / Mutilate the ORS or this booklet.
8. Do not break the seals of the question – paper booklet before instructed to do so by the invigilators.

B. Filling the bottom-half of the ORS:

9. The ORS has CODE printed on its lower and upper Parts.
10. Make sure the CODE on the **ORS** is the same as that on this booklet. If the Codes do not match, ask **for a change of the Booklet**.
11. Write your Registration No., Name and Name of centre and sign with pen in appropriate boxes. Do not write these anywhere else.
12. Darken the appropriate bubbles under each digit of your Registration No. with HB Pencil.

C. Question paper format and Marking scheme:

13. The question paper consists of **3 parts** (Chemistry, Mathematics and Physics). Each part consists of **four** Sections.
14. For each question in **Section I**: you will be awarded **5 marks** if you darken **only the bubble** corresponding to the correct answer and **zero** mark if no bubbles are darkened. In all other cases, **minus two (-2) mark** will be awarded.
15. For each question in **Section II**: you will be awarded 3 marks if you darken the bubble corresponding to the correct answer and **zero mark** if no bubbles are darkened. No negative marks will be awarded for incorrect answers in this Section.
16. For each question in **Section III**: you will be **awarded 3 marks** if you darken **only the bubble** corresponding to the correct answer and **zero** mark if no bubbles are darkened. In all other cases, **minus one (-1) mark** will be awarded.
17. For each question in Section IV: you will be awarded 2 marks for each row in which your darkened the bubbles(s) corresponding to the correct answer. Thus each question in this section carries a maximum of 8 marks. There is no negative marks awarded for incorrect answer(s) in this Section.

Write your name, registration number and sign in the space provided on the back page of this booklet.

Useful Data

Atomic Numbers: B 5; C 6; N 7; O 8; F 9; Na 11; Si 14; P 15; S 16; Cl 17; Ti 22;
V 23; Cr 24; Ni 28; Cu 29; Br 35; Rh 45; Sn 50; Xe 54; Tl 81.

1 amu	=	1.66×10^{-27} kg	e	=	1.6×10^{-19} C
R	=	0.082 L-atm K^{-1} mol $^{-1}$	c	=	3.0×10^8 m s $^{-1}$
h	=	6.626×10^{-34} J s	F	=	96500 C mol $^{-1}$
N_A	=	6.022×10^{23}	R_H	=	2.18×10^{-18} J
m_e	=	9.1×10^{-31} kg	$4\pi\epsilon_0$	=	1.11×10^{-10} J $^{-1}$ C 2 m $^{-1}$

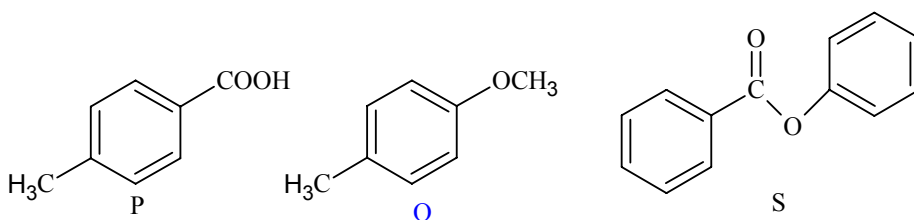
IITJEE 2010 PAPER-2 [Code – 0]

PART - I: CHEMISTRY

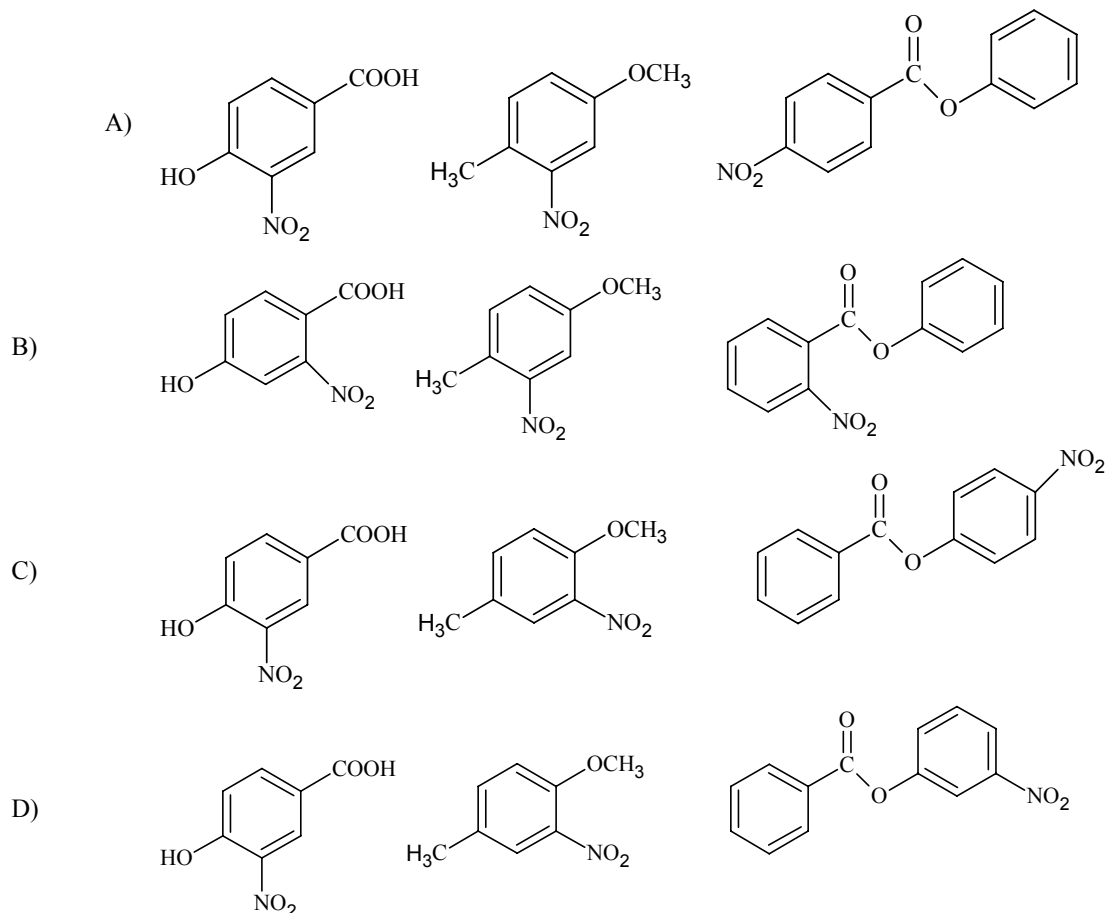
SECTION – I (Single Correct Choice Type)

This Section contains **6 multiple choice questions**. Each question has four choices A), B), C) and D) out of which **ONLY ONE** is correct.

*1. The compounds P, Q and S



were separately subjected to nitration using $\text{HNO}_3/\text{H}_2\text{SO}_4$ mixture. The major product formed in each case respectively, is



SECTION-II (Integer Type)

This Section contains **5 questions**. The answer to each question is a **single-digit integer**, ranging from 0 to 9. The correct digit below the question no. in the **ORS** is to be bubbled.

- *7. Silver (atomic weight = 108 g mol^{-1}) has a density of 10.5 g cm^{-3} . The number of silver atoms on a surface of area 10^{-12} m^2 can be expressed in scientific notation as $y \times 10^x$. The value of x is

Sol. 7

$$d = \frac{\text{mass}}{V} \Rightarrow 10.5 \text{ g/cc means in 1 cc} \Rightarrow 10.5 \text{ g of Ag is present.}$$

$$\text{Number of atoms of Ag in 1 cc} \Rightarrow \frac{10.5}{108} \times N_A$$

$$\text{In 1 cm, number of atoms of Ag} = \sqrt[3]{\frac{10.5}{108} N_A}$$

$$\text{In 1 cm}^2, \text{ number of atoms of Ag} = \left(\frac{10.5}{108} N_A \right)^{2/3}$$

$$\begin{aligned} \text{In } 10^{-12} \text{ m}^2 \text{ or } 10^{-8} \text{ cm}^2, \text{ number of atoms of Ag} &= \left(\frac{10.5}{108} N_A \right)^{2/3} \times 10^{-8} = \left(\frac{1.05 \times 6.022 \times 10^{24}}{108} \right)^{2/3} \times 10^{-8} \\ &= 1.5 \times 10^7 \end{aligned}$$

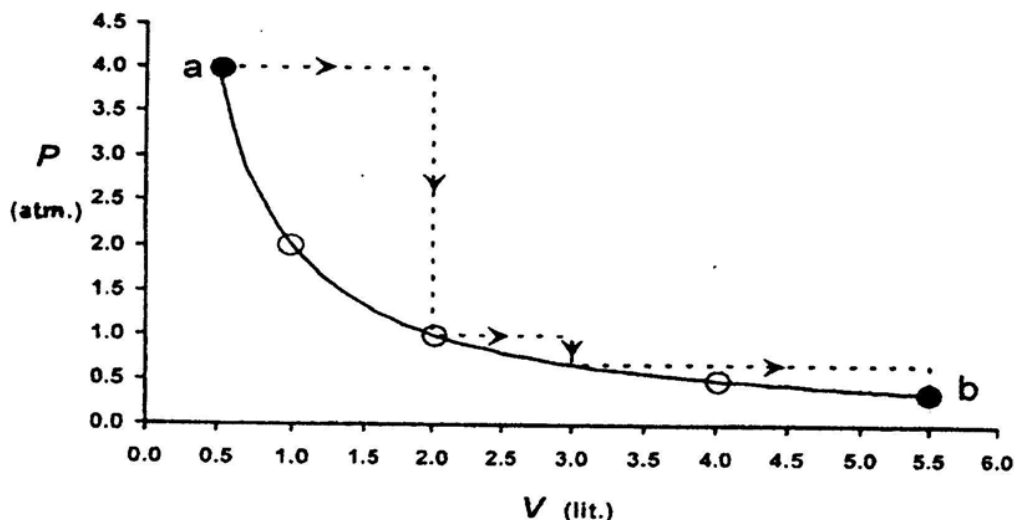
Hence $x = 7$

- *8. Among the following, the number of elements showing only one non-zero oxidation state is O, Cl, F, N, P, Sn, Tl, Na, Ti

Sol. 2

Na, F show only one non-zero oxidation state.

- *9. One mole of an ideal gas is taken from a to b along two paths denoted by the solid and the dashed lines as shown in the graph below. If the work done along the solid line path is w_s and that along the dotted line path is w_d , then the integer closest to the ratio w_d/w_s is



Sol. 2

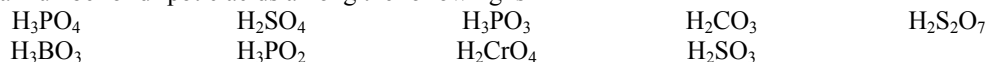
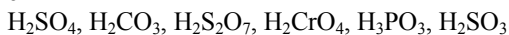
$$w_d = 4 \times 1.5 + 1 \times 1 + 2.5 \times 2/3 = 8.65$$

Process is isothermal

$$w_s = 2 \times 2.303 \log \frac{5.5}{0.5} = 2 \times 2.303 \times \log 11 = 2 \times 2.303 \times 1.0414 = 4.79$$

$$\frac{w_d}{w_s} = \frac{8.65}{4.79} = 1.80 \approx 2$$

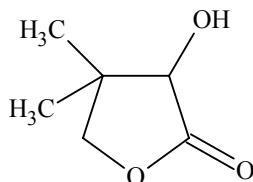
*10. The total number of dirpotic acids among the following is

**Sol.** 611. Total number of geometrical isomers for the complex $[\text{RhCl}(\text{CO})(\text{PPh}_3)(\text{NH}_3)]$ is**Sol.** 3**SECTION-III (Paragraph Type)**

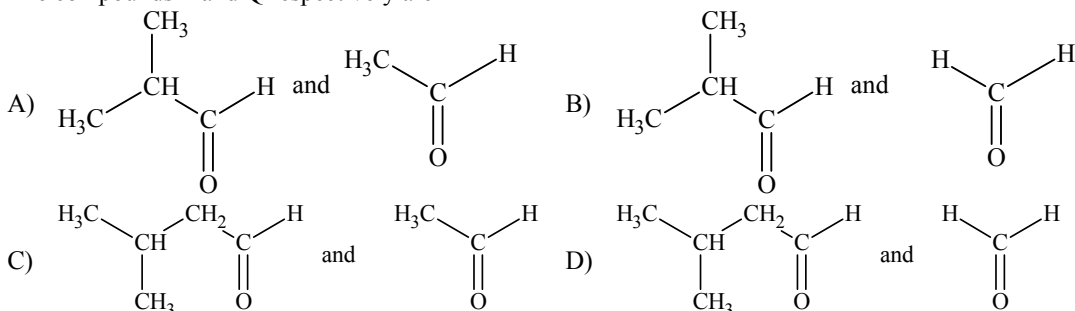
This Section contains 2 paragraphs. Based upon each of the paragraphs 3 multiple choice questions have to be answered. Each of these questions has four choices (A), (B), (C) and (D) out of which **ONLY ONE** is correct.

Paragraph for questions 12 to 14

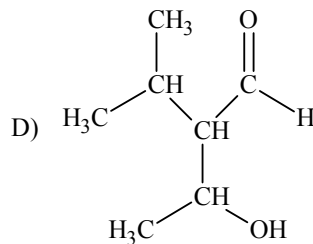
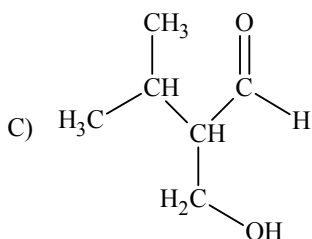
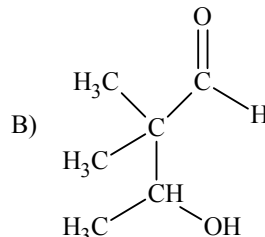
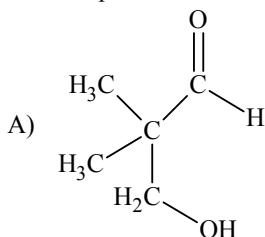
Two aliphatic aldehydes P and Q react in the presence of aqueous K_2CO_3 to give compound R, which upon treatment with HCN provides compound S. On acidification and heating, S gives the product shown below:



12. The compounds P and Q respectively are

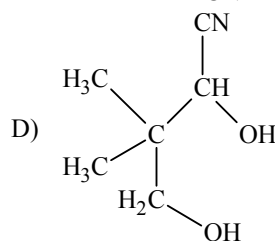
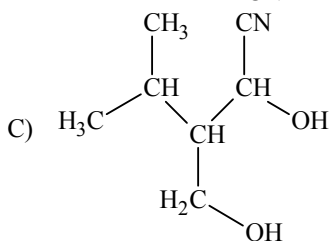
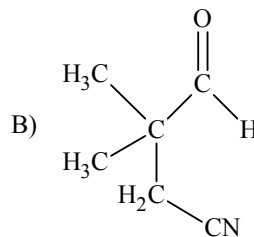
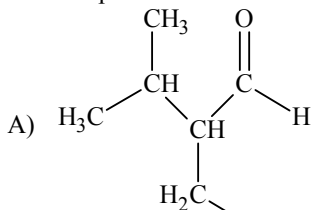
**Sol.** (B)

13. The compound R is



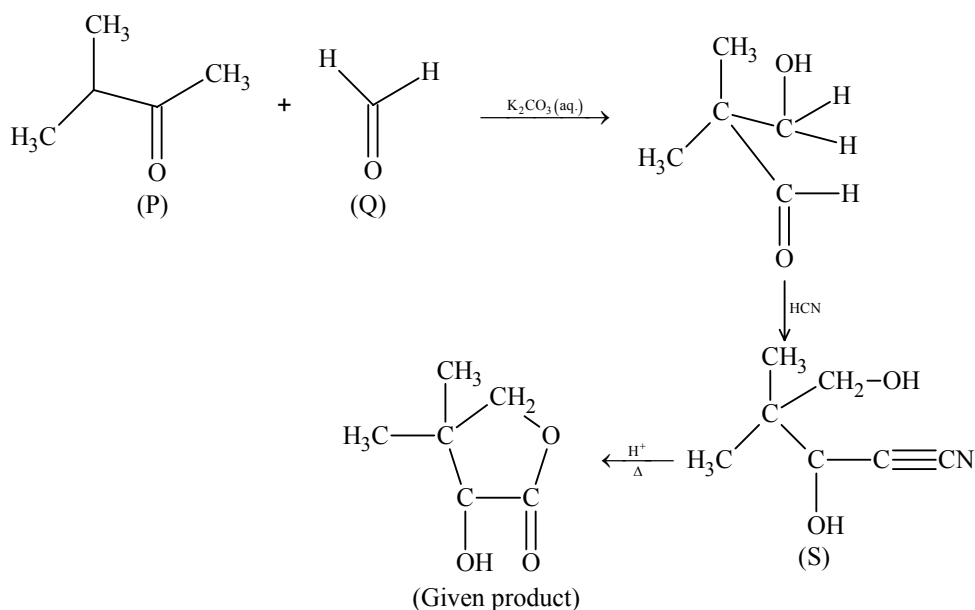
Sol. (A)

14. The compound S is



Sol. (D)

Sol: (12 to 14)



Paragraph for questions 15 to 17

The hydrogen-like species Li^{2+} is in a spherically symmetric state S_1 with one radial node. Upon absorbing light the ion undergoes transition to a state S_2 . The state S_2 has one radial node and its energy is equal to the ground state energy of the hydrogen atom.

- *15. The state S_1 is
- | | |
|-------|-------|
| A) 1s | B) 2s |
| C) 2p | D) 3s |

Sol. (B)
 For, S_1 (spherically symmetrical)
 node = 1
 $\Rightarrow n - 1 = 1$
 $n = 2$
 For S_2 , radial node = 1
 $E_{S_2} = \frac{-13.6 \times Z^2}{n^2} = E_H$ in ground state = -13.6
 $E = \frac{-13.6 \times 9}{n^2} \Rightarrow n = 3$
 So, state S_1 is 2s and S_2 is 3p.

- *16. Energy of the state S_1 in units of the hydrogen atom ground state energy is
- | | |
|---------|---------|
| A) 0.75 | B) 1.50 |
| C) 2.25 | D) 4.50 |

Sol. (C)

$$\frac{E_{S_1}}{E_{H(\text{ground})}} = \frac{-13.6 \times 9}{4 \times (-13.6)} = 2.25$$

- *17. The orbital angular momentum quantum number of the state S_2 is
- | | |
|------|------|
| A) 0 | B) 1 |
| C) 2 | D) 3 |

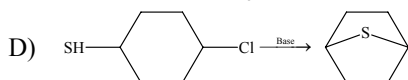
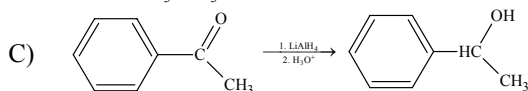
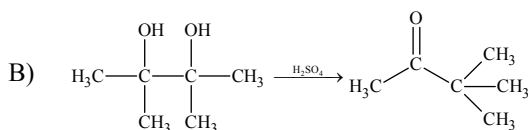
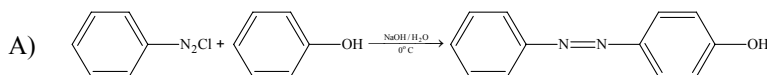
Sol. (B)
 Azimuthal quantum number for $S_2 = \ell = 1$

SECTION-IV (Matrix Type)

This Section contains **2 questions**. Each question has **four statements** (A, B, C and D) given in **Column I** and five statements (p, q, r, s and t) in **Column II**. Any given statement in **Column I** can have correct matching with one or more statement(s) given in **Column II**. For example, if for a given question, statement B matches with the statements given in q and r, then for that particular question, against statement B, darken the bubbles corresponding to q and r in the **ORS**.

18. Match the reactions in Column I with appropriate options in Column II.

Column I



Column II

p) Racemic mixture

q) Addition reaction

r) Substitution reaction

s) Coupling reaction

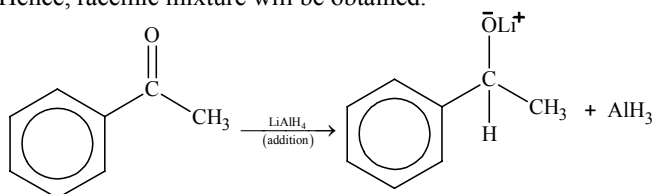
t) Carbocation intermediate

Sol. (A – r, s, t); (B – t); (C – p, q); (D – r)

(A) It is an example of electrophilic substitution reaction which results in coupled product hence it is coupling reaction also.

(B) Pinacole-pinacolone rearrangement. In this reaction intermediate is carbocation.

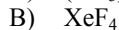
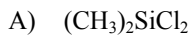
(C) It is an example of addition reaction by carbonyl compounds and both enantiomers will be formed. Hence, racemic mixture will be obtained.



(D) It is an example of nucleophilic substitution.

19. All the compounds listed in Column I react with water. Match the result of the respective reactions with the appropriate options listed in Column II.

Column I



Column II

p) Hydrogen halide formation

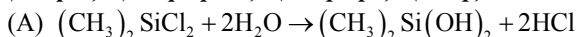
q) Redox reaction

r) Reacts with glass

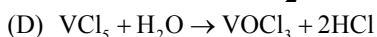
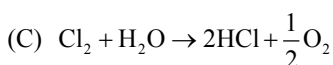
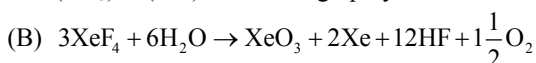
s) Polymerization

t) O_2 formation

Sol. (A – p, s); (B – p, q, r, t); (C – p, q, t); (D – p)



$(\text{CH}_3)_2\text{Si}(\text{OH})_2$ can undergo polymerization to form silicones.



PART - II: MATHEMATICS

SECTION – I (Single Correct Choice Type)

This Section contains **6 multiple choice questions**. Each question has four choices A), B), C) and D) out of which **ONLY ONE** is correct.

20. If the distance of the point P (1, -2, 1) from the plane $x + 2y - 2z = \alpha$, where $\alpha > 0$, is 5, then the foot of the perpendicular from P to the plane is

(A) $\left(\frac{8}{3}, \frac{4}{3}, -\frac{7}{3}\right)$ (B) $\left(\frac{4}{3}, -\frac{4}{3}, \frac{1}{3}\right)$
 (C) $\left(\frac{1}{3}, \frac{2}{3}, \frac{10}{3}\right)$ (D) $\left(\frac{2}{3}, -\frac{1}{3}, \frac{5}{2}\right)$

Sol. (A)

Distance of point (1, -2, 1) from plane $x + 2y - 2z = \alpha$ is $5 \Rightarrow \alpha = 10$.

Equation of PQ $\frac{x-1}{1} = \frac{y+2}{2} = \frac{z-1}{-2} = t$

$Q \equiv (t+1, 2t-2, -2t+1)$ and $PQ = 5 \Rightarrow t = \frac{5+\alpha}{9} = \frac{5}{3} \Rightarrow Q \equiv \left(\frac{8}{3}, \frac{4}{3}, -\frac{7}{3}\right)$.

21. A signal which can be green or red with probability $\frac{4}{5}$ and $\frac{1}{5}$ respectively, is received by station A and then transmitted to station B. The probability of each station receiving the signal correctly is $\frac{3}{4}$. If the signal received at station B is green, then the probability that the original signal was green is

(A) $\frac{3}{5}$ (B) $\frac{6}{7}$
 (C) $\frac{20}{23}$ (D) $\frac{9}{20}$

Sol. (C)

Event G = original signal is green

E_1 = A receives the signal correct

E_2 = B receives the signal correct

E = signal received by B is green

$P(\text{signal received by B is green}) = P(GE_1E_2) + P(G\bar{E}_1\bar{E}_2) + P(\bar{G}E_1\bar{E}_2) + P(\bar{G}\bar{E}_1E_2)$

$P(E) = \frac{46}{5 \times 16}$

$P(G/E) = \frac{40/5 \times 16}{46/5 \times 16} = \frac{20}{23}$.

22. Two adjacent sides of a parallelogram ABCD are given by $\overline{AB} = 2\hat{i} + 10\hat{j} + 11\hat{k}$ and $\overline{AD} = -\hat{i} + 2\hat{j} + 2\hat{k}$. The side AD is rotated by an acute angle α in the plane of the parallelogram so that AD becomes AD'. If AD' makes a right angle with the side AB, then the cosine of the angle α is given by

(A) $\frac{8}{9}$ (B) $\frac{\sqrt{17}}{9}$
 (C) $\frac{1}{9}$ (D) $\frac{4\sqrt{5}}{9}$

Sol. (B)

$$\overline{AD} = \overline{AB} \times (\overline{AB} \times \overline{AD}) = 5(6\hat{i} - 10\hat{j} - 2\hat{k}) \Rightarrow \cos\alpha = \frac{|\overline{AD}' \cdot \overline{AD}|}{|\overline{AD}'||\overline{AD}|} = \frac{\sqrt{17}}{9}.$$

*23. For $r = 0, 1, \dots, 10$, let A_r , B_r and C_r denote, respectively, the coefficient of x^r in the expansions of $(1+x)^{10}$, $(1+x)^{20}$ and $(1+x)^{30}$. Then $\sum_{r=1}^{10} A_r (B_{10}B_r - C_{10}A_r)$ is equal to

- (A) $B_{10} - C_{10}$ (B) $A_{10} (B_{10}^2 - C_{10}A_{10})$
 (C) 0 (D) $C_{10} - B_{10}$

Sol. (D)

$$\text{Let } y = \sum_{r=1}^{10} A_r (B_{10}B_r - C_{10}A_r)$$

$$\sum_{r=1}^{10} A_r B_r = \text{coefficient of } x^{20} \text{ in } ((1+x)^{10} (x+1)^{20}) - 1$$

$$= C_{20} - 1 = C_{10} - 1 \text{ and } \sum_{r=1}^{10} (A_r)^2 = \text{coefficient of } x^{10} \text{ in } ((1+x)^{10} (x+1)^{10}) - 1 = B_{10} - 1$$

$$\Rightarrow y = B_{10}(C_{10} - 1) - C_{10}(B_{10} - 1) = C_{10} - B_{10}.$$

24. Let f be a real-valued function defined on the interval $(-1, 1)$ such that $e^{-x} f(x) = 2 + \int_0^x \sqrt{t^4 + 1} dt$, for all x

$\in (-1, 1)$ and let f^{-1} be the inverse function of f . Then $(f^{-1})'(2)$ is equal to

- (A) 1 (B) $1/3$
 (C) $1/2$ (D) $1/e$

Sol. (B)

$$e^{-x} f(x) = 2 + \int_0^x \sqrt{t^4 + 1} dt \quad \dots(i)$$

$$f(f^{-1}(x)) = x$$

$$\Rightarrow f'(f^{-1}(x)) (f^{-1}(x))' = 1 \Rightarrow (f^{-1}(2))' = \frac{1}{f'(f^{-1}(2))} \Rightarrow f(0) = 2 \Rightarrow f^{-1}(2) = 0$$

$$(f^{-1}(2))' = \frac{1}{f'(0)}$$

$$e^{-x}(f(x) - f(x)) = \sqrt{x^4 + 1}$$

$$\text{Put } x = 0 \Rightarrow f'(0) - 2 = 1 \Rightarrow f'(0) = 3$$

$$(f^{-1}(2))' = 1/3.$$

*25. Let $S = \{1, 2, 3, 4\}$. The total number of unordered pairs of disjoint subsets of S is equal to

- (A) 25 (B) 34
 (C) 42 (D) 41

Sol. (D)

Total number of unordered pairs of disjoint subsets

$$= \frac{3^4 + 1}{2} = 41.$$

SECTION – II (Integer Type)

This Section contains **5 questions**. The answer to each question is a **single-digit integer**, ranging from 0 to 9. The correct digit below the question no. in the **ORS** is to be bubbled.

- *26. Let $a_1, a_2, a_3, \dots, a_{11}$ be real numbers satisfying $a_1 = 15, 27 - 2a_2 > 0$ and $a_k = 2a_{k-1} - a_{k-2}$ for $k = 3, 4, \dots, 11$. If $\frac{a_1^2 + a_2^2 + \dots + a_{11}^2}{11} = 90$, then the value of $\frac{a_1 + a_2 + \dots + a_{11}}{11}$ is equal to

Sol. (0)

$a_k = 2a_{k-1} - a_{k-2} \Rightarrow a_1, a_2, \dots, a_{11}$ are in A.P.

$$\therefore \frac{a_1^2 + a_2^2 + \dots + a_{11}^2}{11} = \frac{11a^2 + 35 \times 11d^2 + 10ad}{11} = 90$$

$$\Rightarrow 225 + 35d^2 + 150d = 90$$

$$35d^2 + 150d + 135 = 0 \Rightarrow d = -3, -9/7$$

$$\text{Given } a_2 < \frac{27}{2} \therefore d = -3 \text{ and } d \neq -9/7 \Rightarrow \frac{a_1 + a_2 + \dots + a_{11}}{11} = \frac{11}{2} [30 - 10 \times 3] = 0.$$

27. Let f be a function defined on \mathbb{R} (the set of all real numbers) such that $f'(x) = 2010(x - 2009)(x - 2010)^2(x - 2011)^3(x - 2012)^4$, for all $x \in \mathbb{R}$. If g is a function defined on \mathbb{R} with values in the interval $(0, \infty)$ such that $f(x) = \ln(g(x))$, for all $x \in \mathbb{R}$, then the number of points in \mathbb{R} at which g has a local maximum is

Sol. (1)

$$f(x) = \ln\{g(x)\}$$

$$g(x) = e^{f(x)}$$

$$g'(x) = e^{f(x)} \cdot f'(x)$$

$$g'(x) = 0 \Rightarrow f'(x) = 0 \text{ as } e^{f(x)} \neq 0$$

$$\Rightarrow 2010(x - 2009)(x - 2010)^2(x - 2011)^3(x - 2012)^4 = 0$$

so there is only one point of local maxima.

28. Let k be a positive real number and let $A = \begin{bmatrix} 2k-1 & 2\sqrt{k} & 2\sqrt{k} \\ 2\sqrt{k} & 1 & -2k \\ -2\sqrt{k} & 2k & -1 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 2k-1 & \sqrt{k} \\ 1-2k & 0 & 2\sqrt{k} \\ -\sqrt{k} & -2\sqrt{k} & 0 \end{bmatrix}$. If \det

$(\text{adj } A) + \det(\text{adj } B) = 10^6$, then $[k]$ is equal to

[Note : $\text{adj } M$ denotes the adjoint of a square matrix M and $[k]$ denotes the largest integer less than or equal to k].

Sol. (5)

$$|A| = (2k + 1)^3, |B| = 0 \quad (\text{Since } B \text{ is a skew-symmetric matrix of order } 3)$$

$$\Rightarrow \det(\text{adj } A) = |A|^{n-1} = ((2k + 1)^3)^2 = 106 \Rightarrow 2k + 1 = 10 \Rightarrow 2k = 9$$

$$[k] = 4.$$

- *29. Two parallel chords of a circle of radius 2 are at a distance $\sqrt{3} + 1$ apart. If the chords subtend at the centre, angles of $\frac{\pi}{k}$ and $\frac{2\pi}{k}$, where $k > 0$, then the value of $[k]$ is

[Note : $[k]$ denotes the largest integer less than or equal to k].

Sol. (2)

$$2 \cos \frac{\pi}{2k} + 2 \cos \frac{\pi}{k} = \sqrt{3} + 1$$

$$\cos \frac{\pi}{2k} + \cos \frac{\pi}{k} = \frac{\sqrt{3}+1}{2}$$

$$\text{Let } \frac{\pi}{k} = \theta, \cos \theta + \cos \frac{\theta}{2} = \frac{\sqrt{3}+1}{2} \Rightarrow 2 \cos^2 \frac{\theta}{2} - 1 + \cos \frac{\theta}{2} = \frac{\sqrt{3}+1}{2}$$

$$\cos \frac{\theta}{2} = t \quad 2t^2 + t - \frac{\sqrt{3}+3}{2} = 0$$

$$t = \frac{-1 \pm \sqrt{1+4(3+\sqrt{3})}}{4} = \frac{-1 \pm (2\sqrt{3}+1)}{4} = \frac{-2-2\sqrt{3}}{4}, \frac{\sqrt{3}}{2} \quad \because t \in [-1, 1], \cos \frac{\theta}{2} = \frac{\sqrt{3}}{2}$$

$$\frac{\theta}{2} = \frac{\pi}{6} \Rightarrow k = 3.$$

- *30. Consider a triangle ABC and let a, b and c denote the lengths of the sides opposite to vertices A, B and C respectively. Suppose a = 6, b = 10 and the area of the triangle is $15\sqrt{3}$. If $\angle ACB$ is obtuse and if r denotes the radius of the incircle of the triangle, then r^2 is equal to

Sol. (3)

$$\Delta = \frac{1}{2} ab \sin C \Rightarrow \sin C = \frac{2\Delta}{ab} = \frac{2 \times 15\sqrt{3}}{6 \times 10} = \frac{\sqrt{3}}{2} \Rightarrow C = 120^\circ$$

$$\Rightarrow c = \sqrt{a^2 + b^2 - 2ab \cos C} \\ = \sqrt{6^2 + 10^2 - 2 \times 6 \times 10 \times \cos 120^\circ} = 14$$

$$\therefore r = \frac{\Delta}{s} \Rightarrow r^2 = \frac{225 \times 3}{\left(\frac{6+10+14}{2}\right)^2} = 3.$$

SECTION – III (Paragraph Type)

This Section contains **2 paragraphs**. Based upon each of the paragraphs **3 multiple choice questions** have to be answered. Each of these questions has four choices (A), (B), (C) and (D) out of which **ONLY ONE** is correct.

Paragraph for questions 31 to 33.

Consider the polynomial $f(x) = 1 + 2x + 3x^2 + 4x^3$. Let s be the sum of all distinct real roots of $f(x)$ and let $t = |s|$.

31. The real number s lies in the interval

(A) $\left(-\frac{1}{4}, 0\right)$ (B) $\left(-11, -\frac{3}{4}\right)$
 (C) $\left(-\frac{3}{4}, -\frac{1}{2}\right)$ (D) $\left(0, \frac{1}{4}\right)$

Sol. (C)

$$\text{Since, } f\left(-\frac{1}{2}\right) \cdot f\left(-\frac{3}{4}\right) < 0 \Rightarrow S \text{ lie in } \left(-\frac{3}{4}, -\frac{1}{2}\right).$$

32. The area bounded by the curve $y = f(x)$ and the lines $x = 0$, $y = 0$ and $x = t$, lies in the interval

(A) $\left(\frac{3}{4}, 3\right)$ (B) $\left(\frac{21}{64}, \frac{11}{16}\right)$
 (C) (9, 10) (D) $\left(0, \frac{21}{64}\right)$

Sol. (A)

$$-\frac{3}{4} < s < -\frac{1}{2}$$

$$\frac{1}{2} < t < \frac{3}{4}$$

$$\int_0^{1/2} (4x^3 + 3x^2 + 2x + 1) dx < \text{area} < \int_0^{3/4} (4x^3 + 3x^2 + 2x + 1) dx$$

$$[x^4 + x^3 + x^2 + x]_0^{1/2} < \text{area} < [x^4 + x^3 + x^2 + x]_0^{3/4}$$

$$\frac{1}{16} + \frac{1}{8} + \frac{1}{4} + \frac{1}{2} < \text{area} < \frac{81}{256} + \frac{27}{64} + \frac{9}{16} + \frac{3}{4}$$

$$\frac{15}{16} < \text{area} < \frac{525}{256}$$

33. The function $f'(x)$ is(A) increasing in $\left(-t, -\frac{1}{4}\right)$ and decreasing in $\left(-\frac{1}{4}, t\right)$ (B) decreasing in $\left(-t, -\frac{1}{4}\right)$ and increasing in $\left(-\frac{1}{4}, t\right)$ (C) increasing in $(-t, t)$ (D) decreasing in $(-t, t)$ **Sol. (B)**

$$f''(x) = 2[12x + 3] = 0 \Rightarrow x = -1/4.$$

Paragraph for questions 34 to 36.Tangents are drawn from the point $P(3, 4)$ to the ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$ touching the ellipse at points A and B.

*34. The coordinates of A and B are

(A) $(3, 0)$ and $(0, 2)$ (B) $\left(-\frac{8}{5}, \frac{2\sqrt{161}}{15}\right)$ and $\left(-\frac{9}{5}, \frac{8}{5}\right)$ (C) $\left(-\frac{8}{5}, \frac{2\sqrt{161}}{15}\right)$ and $(0, 2)$ (D) $(3, 0)$ and $\left(-\frac{9}{5}, \frac{8}{5}\right)$ **Sol. (D)**

$$y = mx + \sqrt{9m^2 + 4}$$

$$4 - 3m = \sqrt{9m^2 + 4}$$

$$16 + 9m^2 - 24m = 9m^2 + 4 \Rightarrow m = \frac{12}{24} = \frac{1}{2}$$

$$\text{Equation is } y - 4 = \frac{1}{2}(x - 3)$$

$$2y - 8 = x - 3 \Rightarrow x - 2y + 5 = 0$$

$$\text{Let } B = (\alpha, \beta) \Rightarrow \frac{x\alpha}{9} + \frac{y\beta}{4} - 1 = 0 \Rightarrow \frac{\alpha/9}{1} = \frac{\beta/4}{-2} = \frac{-1}{5} \Rightarrow \alpha = -\frac{9}{5}, \beta = \frac{8}{5}$$

$$B \equiv \left(-\frac{9}{5}, \frac{8}{5}\right).$$

*35. The orthocentre of the triangle PAB is

- (A) $\left(5, \frac{8}{7}\right)$ (B) $\left(\frac{7}{5}, \frac{25}{8}\right)$
 (C) $\left(\frac{11}{5}, \frac{8}{5}\right)$ (D) $\left(\frac{8}{25}, \frac{7}{5}\right)$

Sol. (C)

Slope of BD must be 0

$$\Rightarrow y - \frac{8}{5} = 0 \quad \left(x + \frac{9}{5}\right) \Rightarrow y = \frac{8}{5}$$

Hence y coordinate of D is 8/5.

*36. The equation of the locus of the point whose distances from the point P and the line AB are equal, is

- (A) $9x^2 + y^2 - 6xy - 54x - 62y + 241 = 0$ (B) $x^2 + 9y^2 + 6xy - 54x + 62y - 241 = 0$
 (C) $9x^2 + 9y^2 - 6xy - 54x - 62y - 241 = 0$ (D) $x^2 + y^2 - 2xy + 27x + 31y - 120 = 0$

Sol. (A)

Locus is parabola

$$\text{Equation of AB is } \frac{3x}{9} + \frac{4y}{4} = 1 \Rightarrow \frac{x}{3} + y = 1 \Rightarrow x + 3y - 3 = 0$$

$$(x - 3)^2 + (y - 4)^2 = \frac{(x + 3y - 3)^2}{10}$$

$$10x^2 + 90 - 60x + 10y^2 + 160 - 80y = x^2 + 9y^2 + 9 + 6xy - 6x - 18y$$

$$\Rightarrow 9x^2 + y^2 - 6xy - 54x - 62y + 241 = 0.$$

SECTION – IV (Matrix Type)

This Section contains **2 questions**. Each question has **four statements** (A, B, C and D) given in **Column I** and five statements (p, q, r, s and t) in **Column II**. Any given statement in **Column I** can have correct matching with one or more statement(s) given in **Column II**. For example, if for a given question, statement B matches with the statements given in q and r, then for that particular question, against statement B, darken the bubbles corresponding to q and r in the **ORS**.

*37. Match the statements in column-I with those in column-II.

[Note: Here z takes the values in the complex plane and $\text{Im } z$ and $\text{Re } z$ denote, respectively, the imaginary part and the real part of z]

- | Column – I | Column – II |
|--|--|
| (A) The set of points z satisfying $ z - i z = z + i z $ is contained in or equal to | (p) an ellipse with eccentricity $\frac{4}{5}$ |
| (B) The set of points z satisfying $ z + 4 + z - 4 = 10$ is contained in or equal to | (q) the set of points z satisfying $\text{Im } z = 0$ |
| (C) If $ \omega = 2$, then the set of points $z = \omega - 1/\omega$ is contained in or equal to | (r) the set of points z satisfying $ \text{Im } z \leq 1$ |
| (D) If $ \omega = 1$, then the set of points $z = \omega + 1/\omega$ is contained in or equal to | (s) the set of points z satisfying $ \text{Re } z \leq 1$ |
| | (t) the set of points z satisfying $ z \leq 3$ |

Sol. (A) (q)

$$\left| \frac{z}{|z|} - i \right| = \left| \frac{z}{|z|} + i \right|, z \neq 0$$

$\frac{z}{|z|}$ is unimodular complex number

and lies on perpendicular bisector of i and $-i$

$$\Rightarrow \frac{z}{|z|} = \pm 1 \Rightarrow z = \pm 1|z| \Rightarrow z \text{ is real number} \Rightarrow \text{Im}(z) = 0.$$

(B) (p)

$$|z + 4| + |z - 4| = 10$$

z lies on an ellipse whose focus are $(4, 0)$ and $(-4, 0)$ and length of major axis is 10

$$\Rightarrow 2ae = 8 \text{ and } 2a = 10 \Rightarrow e = 4/5$$

$$|\text{Re}(z)| \leq 5.$$

(C) (p), (t)

$$|w| = 2 \Rightarrow w = 2(\cos\theta + i\sin\theta)$$

$$x + iy = 2(\cos\theta + i\sin\theta) - \frac{1}{2}(\cos\theta - i\sin\theta)$$

$$= \frac{3}{2}\cos\theta + i\frac{5}{2}\sin\theta \Rightarrow \frac{x^2}{(3/2)^2} + \frac{y^2}{(5/2)^2} = 1$$

$$e^2 = 1 - \frac{9/4}{25/4} = 1 - \frac{9}{25} = \frac{16}{25} \Rightarrow e = \frac{4}{5}$$

(D) (q), (t)

$$|w| = 1 \Rightarrow x + iy = \cos\theta + i\sin\theta + \cos\theta - i\sin\theta$$

$$x + iy = 2\cos\theta$$

$$|\text{Re}(z)| \leq 1, |\text{Im}(z)| = 0.$$

38. Match the statements in column-I with those in column-II.

Column – I

Column – II

(A) A line from the origin meets the lines $\frac{x-2}{1} = \frac{y-1}{-2} = \frac{z+1}{1}$ and (p)

-4

$\frac{x-\frac{8}{2}}{3} = \frac{y+3}{-1} = \frac{z-1}{1}$ at P and Q respectively. If length PQ = d, then d^2 is

*(B) The values of x satisfying $\tan^{-1}(x+3) - \tan^{-1}(x-3) = \sin^{-1}\left(\frac{3}{5}\right)$ (q) 0

are

(C) Non-zero vectors \vec{a} , \vec{b} and \vec{c} satisfy $\vec{a} \cdot \vec{b} = 0$, (r) 4

$(\vec{b} - \vec{a}) \cdot (\vec{b} + \vec{c}) = 0$ and $2|\vec{b} + \vec{c}| = |\vec{b} - \vec{a}|$. If $\vec{a} = \mu\vec{b} + 4\vec{c}$, then the possible values of μ are

(D) Let f be the function on $[-\pi, \pi]$ given by $f(0) = 9$ and $f(x) =$ (s) 5

$\sin\left(\frac{9x}{2}\right) / \sin\left(\frac{x}{2}\right)$ for $x \neq 0$. The value of $\frac{2}{\pi} \int_{-\pi}^{\pi} f(x) dx$ is

(t) 6

Sol. (A). (t)

Let the line be $\frac{x}{a} = \frac{y}{b} = \frac{z}{c}$ intersects the lines

$$\Rightarrow S.D = 0 \Rightarrow a + 3b + 5c = 0 \text{ and } 3a + b - 5c = 0 \Rightarrow a : b : c :: 5r : -5r : 2r$$

on solving with given lines we get points of intersection $P \equiv (5, -5, 2)$ and $Q \equiv \left(\frac{10}{3}, -\frac{10}{3}, \frac{8}{3}\right)$
 $\Rightarrow PQ^2 = d^2 = 6.$

(B) (p), (r)

$$\tan^{-1}(x+3) - \tan^{-1}(x-3) = \sin^{-1}(3/5)$$

$$\Rightarrow \tan^{-1} \frac{(x+3)-(x-3)}{1+(x^2-9)} = \tan^{-1} \frac{3}{4} \Rightarrow \frac{6}{x^2-8} = \frac{3}{4}$$

$$\therefore x^2 - 8 = 8$$

$$\text{or } x = \pm 4.$$

(C) (q), (s)

As $\vec{a} = \mu\vec{b} + 4\vec{c} \Rightarrow \mu(|\vec{b}|) = -4\vec{b} \cdot \vec{c}$ and $|\vec{b}|^2 = 4\vec{a} \cdot \vec{c}$ and $|\vec{b}|^2 + \vec{b} \cdot \vec{c} - \vec{a} \cdot \vec{c} = 0$
 Again, as $2|\vec{b} + \vec{c}| = |\vec{b} - \vec{a}|$
 Solving and eliminating $\vec{b} \cdot \vec{c}$ and eliminating $|\vec{a}|^2$
 we get $(2\mu^2 - 10\mu)|\vec{b}|^2 = 0 \Rightarrow \mu = 0$ and $5.$

(D) (r)

$$I = \frac{2}{\pi} \int_{-\pi}^{\pi} \frac{\sin 9(x/2)}{\sin(x/2)} dx = \frac{2}{\pi} \times 2 \int_0^{\pi} \frac{\sin 9(x/2)}{\sin(x/2)} dx$$

$$x/2 = \theta \Rightarrow dx = 2d\theta$$

$$x = 0, \theta = 0$$

$$x = \pi, \theta = \pi/2$$

$$I = \frac{8}{\pi} \int_0^{\pi/2} \frac{\sin 9\theta}{\sin \theta} d\theta$$

$$= \frac{8}{\pi} \int_0^{\pi/2} \frac{(\sin 9\theta - \sin 7\theta)}{\sin \theta} + \frac{(\sin 7\theta - \sin 5\theta)}{\sin \theta} + \frac{(\sin 5\theta - \sin 3\theta)}{\sin \theta} + \frac{(\sin 3\theta - \sin \theta)}{\sin \theta} + \frac{\sin \theta}{\sin \theta} d\theta$$

$$= \frac{16}{\pi} \int_0^{\pi/2} (\cos 8\theta + \cos 6\theta + \cos 4\theta + \cos 2\theta + 1) d\theta + \frac{8}{\pi} \int_0^{\pi/2} d\theta$$

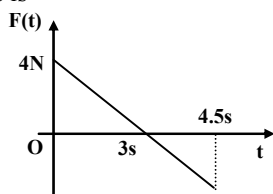
$$= \frac{16}{\pi} \left[\frac{\sin 8\theta}{8} + \frac{\sin 6\theta}{6} + \frac{\sin 4\theta}{4} + \frac{\sin 2\theta}{2} \right] + \frac{8}{\pi} [\theta]_0^{\pi/2} = 0 + \frac{8}{\pi} \times \left[\frac{\pi}{2} - 0 \right] = 4$$

PART - III: PHYSICS

SECTION – I (Single Correct Choice Type)

This Section contains **6 multiple choice questions**. Each question has four choices A), B), C) and D) out of which **ONLY ONE** is correct.

39. A block of mass 2 kg is free to move along the x-axis. It is at rest and from $t = 0$ onwards it is subjected to a time-dependent force $F(t)$ in the x direction. The force $F(t)$ varies with t as shown in the figure. The kinetic energy of the block after 4.5 seconds is



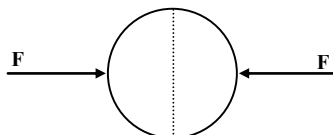
- A) 4.50 J B) 7.50 J C) 5.06 J D) 14.06 J

Sol. (C)

Area under F-t curve = 4.5 kg-m/sec

$$\text{K.E.} = \frac{1}{2}(2)\left(\frac{4.5}{2}\right)^2 = 5.06 \text{ J}$$

40. A uniformly charged thin spherical shell of radius R carries uniform surface charge density of σ per unit area. It is made of two hemispherical shells, held together by pressing them with force F (see figure). F is proportional to



- A) $\frac{1}{\epsilon_0}\sigma^2R^2$ B) $\frac{1}{\epsilon_0}\sigma^2R$ C) $\frac{1}{\epsilon_0}\frac{\sigma^2}{R}$ D) $\frac{1}{\epsilon_0}\frac{\sigma^2}{R^2}$

Sol. (A)

$$\text{Pressure} = \frac{\sigma^2}{2\epsilon_0} \text{ and force} = \frac{\sigma^2}{2\epsilon_0} \times \pi R^2$$

41. A tiny spherical oil drop carrying a net charge q is balanced in still air with a vertical uniform electric field of strength $\frac{81\pi}{7} \times 10^5 \text{ Vm}^{-1}$. When the field is switched off, the drop is observed to fall with terminal velocity $2 \times 10^{-3} \text{ ms}^{-1}$. Given $g = 9.8 \text{ ms}^{-2}$, viscosity of the air = $1.8 \times 10^{-5} \text{ N s m}^{-2}$ and the density of oil = 900 kg m^{-3} , the magnitude of q is
- A) $1.6 \times 10^{-19} \text{ C}$ B) $3.2 \times 10^{-19} \text{ C}$ C) $4.8 \times 10^{-19} \text{ C}$ D) $8.0 \times 10^{-19} \text{ C}$

Sol. (D)

$$\frac{4}{3}\pi R^3 \rho g = qE = 6\pi\eta R v_T$$

$$\therefore q = 8.0 \times 10^{-19} \text{ C}$$

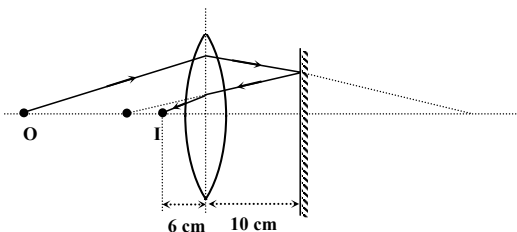
42. A Vernier calipers has 1 mm marks on the main scale. It has 20 equal divisions on the Vernier scale which match with 16 main scale divisions. For this Vernier calipers, the least count is
 A) 0.02 mm B) 0.05 mm C) 0.1 mm D) 0.2 mm

Sol. (D)

$$\begin{aligned} \text{L.C.} &= 1 \text{ M.S.D} - 1 \text{ V.S.D} \\ &= \left(1 - \frac{16}{20}\right) \text{M.S.D} \\ &= \left(1 - \frac{4}{5}\right) (1\text{mm}) = 0.2 \text{ mm} \end{aligned}$$

43. A biconvex lens of focal length 15 cm is in front of a plane mirror. The distance between the lens and the mirror is 10 cm. A small object is kept at a distance of 30 cm from the lens. The final image is
 A) virtual and at a distance of 16 cm from the mirror
 B) real and at a distance of 16 cm from the mirror
 C) virtual and at a distance of 20 cm from the mirror
 D) real and at a distance of 20 cm from the mirror

Sol. (B)



44. A hollow pipe of length 0.8 m is closed at one end. At its open end a 0.5 m long uniform string is vibrating in its second harmonic and it resonates with the fundamental frequency of the pipe. If the tension in the wire is 50 N and the speed of sound is 320 ms^{-1} , the mass of the string is
 A) 5 grams B) 10 grams C) 20 grams D) 40 grams

Sol. (B)

$$\begin{aligned} \frac{v_s}{4L_p} &= \frac{2\sqrt{\frac{T}{\mu}}}{2\ell_s} \\ \mu \ell_s &= 10 \text{ gm} \end{aligned}$$

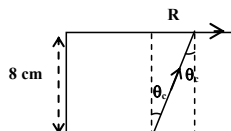
SECTION –II (Integer Type)

This Section contains **5 questions**. The answer to each question is a **single-digit integer**, ranging from 0 to 9. The correct digit below the question no. in the **ORS** is to be bubbled.

45. A large glass slab ($\mu = 5/3$) of thickness 8 cm is placed over a point source of light on a plane surface. It is seen that light emerges out of the top surface of the slab from a circular area of radius R cm. What is the value of R?

Sol. 6

$$\begin{aligned} \sin \theta_c &= 3/5 \\ \therefore R &= 6 \text{ cm.} \end{aligned}$$



46. Image of an object approaching a convex mirror of radius of curvature 20 m along its optical axis is observed to move from $\frac{25}{3}$ m to $\frac{50}{7}$ m in 30 seconds. What is the speed of the object in km per hour ?

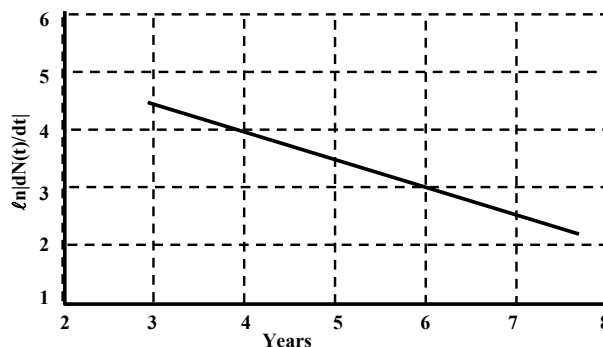
Sol. 3

$$\text{For } v_1 = \frac{50}{7} \text{ m, } u_1 = -25 \text{ m}$$

$$v_2 = \frac{25}{3} \text{ m, } u_2 = -50 \text{ m}$$

$$\text{Speed of object} = \frac{25}{30} \times \frac{18}{5} = 3 \text{ kmph.}$$

47. To determine the half life of a radioactive element, a student plots a graph of $\ln \left| \frac{dN(t)}{dt} \right|$ versus t. Here $\frac{dN(t)}{dt}$ is the rate of radioactive decay at time t. If the number of radioactive nuclei of this element decreases by a factor of p after 4.16 years, the value of p is



Sol. 8

$$N = N_0 e^{-\lambda t}$$

$$\ln |dN/dt| = \ln(N_0 \lambda) - \lambda t$$

$$\text{From graph, } \lambda = \frac{1}{2} \text{ per year}$$

$$t_{1/2} = \frac{0.693}{1/2} = 1.386 \text{ year}$$

$$4.16 \text{ yrs} = 3t_{1/2}$$

$$\therefore p = 8$$

48. A diatomic ideal gas is compressed adiabatically to $\frac{1}{32}$ of its initial volume. If the initial temperature of the gas is T_i (in Kelvin) and the final temperature is aT_i , the value of a is

Sol. 4

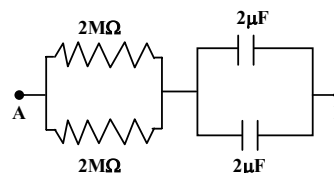
$$TV^{\gamma-1} = \text{constant}$$

$$TV^{7/5-1} = aT \left(\frac{V}{32} \right)^{7/5-1}$$

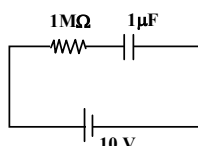
$$\therefore a = 4.$$

49. At time $t = 0$, a battery of 10 V is connected across points A and B in the given circuit. If the capacitors have no charge initially, at what time (in seconds) does the voltage across them becomes 4 volt?

[take $\ln 5 = 1.6$, $\ln 3 = 1.1$]



- Sol.** 2
 $4 = 10(1 - e^{-t/4})$
 $\therefore t = 2 \text{ sec.}$



SECTION –III (Paragraph Type)

This Section contains **2 paragraphs**. Based upon each of the paragraphs **3 multiple choice questions** have to be answered. Each of these questions has four choices (A), (B), (C) and (D) out of which **ONLY ONE** is correct.

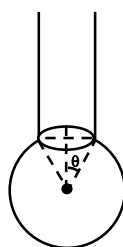
Paragraphs for Question 50 To 52

When liquid medicine of density ρ is to be put in the eye, it is done with the help of a dropper. As the bulb on the top of the dropper is pressed, a drop forms at the opening of the dropper. We wish to estimate the size of the drop. We first assume that the drop formed at the opening is spherical because that requires a minimum increase in its surface energy. To determine the size, we calculate the net vertical force due to the surface tension T when the radius of the drop is R . When the force becomes smaller than the weight of the drop, the drop gets detached from the dropper.

50. If the radius of the opening of the dropper is r , the vertical force due to the surface tension on the drop of radius R (assuming $r \ll R$) is

- A) $2\pi rT$ B) $2\pi RT$ C) $\frac{2\pi r^2 T}{R}$ D) $\frac{2\pi R^2 T}{r}$

- Sol.** (C)
 Surface Tension force = $2\pi r T \frac{r}{R} = \frac{2\pi r^2 T}{R}$



51. If $r = 5 \times 10^{-4} \text{ m}$, $\rho = 10^3 \text{ kg m}^{-3}$, $g = 10 \text{ m/s}^2$, $T = 0.11 \text{ Nm}^{-1}$, the radius of the drop when it detaches from the dropper is approximately

- A) $1.4 \times 10^{-3} \text{ m}$ B) $3.3 \times 10^{-3} \text{ m}$ C) $2.0 \times 10^{-3} \text{ m}$ D) $4.1 \times 10^{-3} \text{ m}$

- Sol.** (A)
 $\frac{2\pi r^2 T}{R} = mg = \frac{4}{3} \pi R^3 \rho g$

52. After the drop detaches, its surface energy is
 A) 1.4×10^{-6} J B) 2.7×10^{-6} J C) 5.4×10^{-6} J D) 8.1×10^{-6} J

Sol. (B)

$$\text{Surface energy} = T(4\pi R^2) = 2.7 \times 10^{-6} \text{ J}$$

Paragraph for questions 53 to 55.

The key feature of Bohr's theory of spectrum of hydrogen atom is the quantization of angular momentum when an electron is revolving around a proton. We will extend this to a general rotational motion to find quantized rotational energy of a diatomic molecule assuming it to be rigid. The rule to be applied is Bohr's quantization condition.

53. A diatomic molecule has moment of inertia I . By Bohr's quantization condition its rotational energy in the n^{th} level ($n = 0$ is not allowed) is
 A) $\frac{1}{n^2} \left(\frac{h^2}{8\pi^2 I} \right)$ B) $\frac{1}{n} \left(\frac{h^2}{8\pi^2 I} \right)$ C) $n \left(\frac{h^2}{8\pi^2 I} \right)$ D) $n^2 \left(\frac{h^2}{8\pi^2 I} \right)$

Sol. (D)

$$L = \frac{nh}{2\pi}$$

$$\text{K.E.} = \frac{L^2}{2I} = \left(\frac{nh}{2\pi} \right)^2 \frac{1}{2I}$$

54. It is found that the excitation frequency from ground to the first excited state of rotation for the CO molecule is close to $\frac{4}{\pi} \times 10^{11}$ Hz. Then the moment of inertia of CO molecule about its centre of mass is close to (Take $h = 2\pi \times 10^{-34}$ Js)
 A) 2.76×10^{-46} kg m² B) 1.87×10^{-46} kg m² C) 4.67×10^{-47} kg m² D) 1.17×10^{-47} kg m²

Sol. (B)

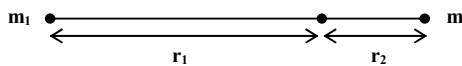
$$h\nu = k.E_{n=2} - k.E_{n=1}$$

$$I = 1.87 \times 10^{-46} \text{ kg m}^2$$

55. In a CO molecule, the distance between C (mass = 12 a.m.u) and O (mass = 16 a.m.u.), where 1 a.m.u. = $\frac{5}{3} \times 10^{-27}$ kg, is close to
 A) 2.4×10^{-10} m B) 1.9×10^{-10} m C) 1.3×10^{-10} m D) 4.4×10^{-11} m

Sol. (C)

$$r_1 = \frac{m_2 d}{m_1 + m_2} \quad \text{and} \quad r_2 = \frac{m_1 d}{m_1 + m_2}$$



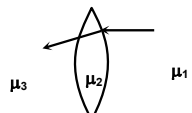
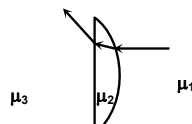
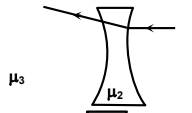
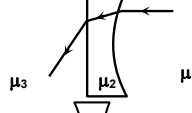

$$I = m_1 r_1^2 + m_2 r_2^2$$

$$\therefore d = 1.3 \times 10^{-10} \text{ m.}$$

SECTION – IV (Matrix Type)

This Section contains **2 questions**. Each question has **four statements** (A, B, C and D) given in **Column I** and five statements (p, q, r, s and t) in **Column II**. Any given statement in **Column I** can have correct matching with one or more statement(s) given in **Column II**. For example, if for a given question, statement B matches with the statements given in q and r, then for that particular question, against statement B, darken the bubbles corresponding to q and r in the **ORS**.

56. Two transparent media of refractive indices μ_1 and μ_3 have a solid lens shaped transparent material of refractive index μ_2 between them as shown in figures in Column II. A ray traversing these media is also shown in the figures. In Column I different relationships between μ_1 , μ_2 and μ_3 are given. Match them to the ray diagram shown in Column II.

Column I	Column II
A) $\mu_1 < \mu_2$	p) 
B) $\mu_1 > \mu_2$	q) 
C) $\mu_2 = \mu_3$	r) 
D) $\mu_2 > \mu_3$	s) 
	t) 

Sol. (A) \rightarrow (p, r), (B) \rightarrow (q, s, t), (C) \rightarrow (p, r, t), (D) \rightarrow (q, s)

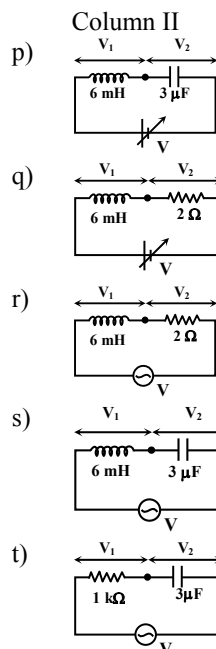
57. You are given many resistances, capacitors and inductors. These are connected to a variable DC voltage source (the first two circuits) or an AC voltage source of 50 Hz frequency (the next three circuits) in different ways as shown in Column II. When a current I (steady state for DC or rms for AC) flows through the circuit, the corresponding voltage V_1 and V_2 . (indicated in circuits) are related as shown in Column I. Match the two

Column I
A) $I \neq 0, V_1$ is proportional to I

B) $I \neq 0, V_2 > V_1$

C) $V_1 = 0, V_2 = V$

D) $I \neq 0, V_2$ is proportional to I



Sol. (A) $\rightarrow (r, s, t)$, (B) $\rightarrow (q, r, s, t)$, (C) $\rightarrow (p, q)$, (D) $\rightarrow (q, r, s, t)$
